Some mathematical and conceptual tools

Modelling wind and dispersion in urban environments

Text book references: And Introduction to Boundary Meteorology, R.B. Stull, chapter 1

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Learning Objectives:

After this class, you will be able to:

- Apply mean averaging rules
- Understand statistical mean, variance and correlation concepts

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Activities during the class:

During this class, you will perform the following activities:

1. Think-pair-share
2. Averaging rule exercise
3. One-minute notes (the muddiest point)

NO summative assignments

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Outline of the class:

1. Turbulence and its spectrum

2. Mean and turbulent parts

3. Some basic statistical methods

4. Turbulent kinetic energy

5. Flux and stress

6. Wrap-up

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Application real data

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Turbulence and its spectrum?

Because turbulence is an intrinsic part of the ABL, we need to study it

- Unresolved problem
- Extremely random

The randomness makes the deterministic description of turbulence difficult so we are forced to use statistics

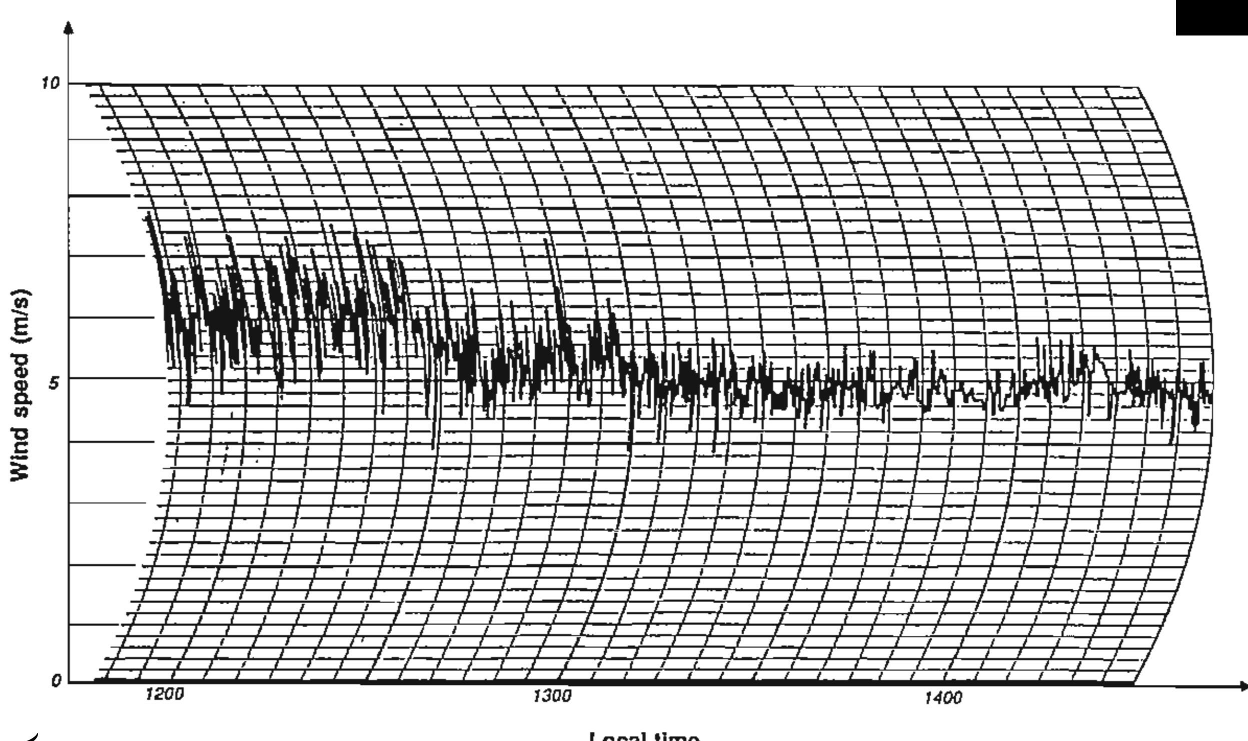

Fig. 2.1 Trace of wind speed obseved in early afternoon.

The **ABL is intrinsically turbulent**, look at a signal of wind speed near-surface for 2.5h

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Think-pair-share

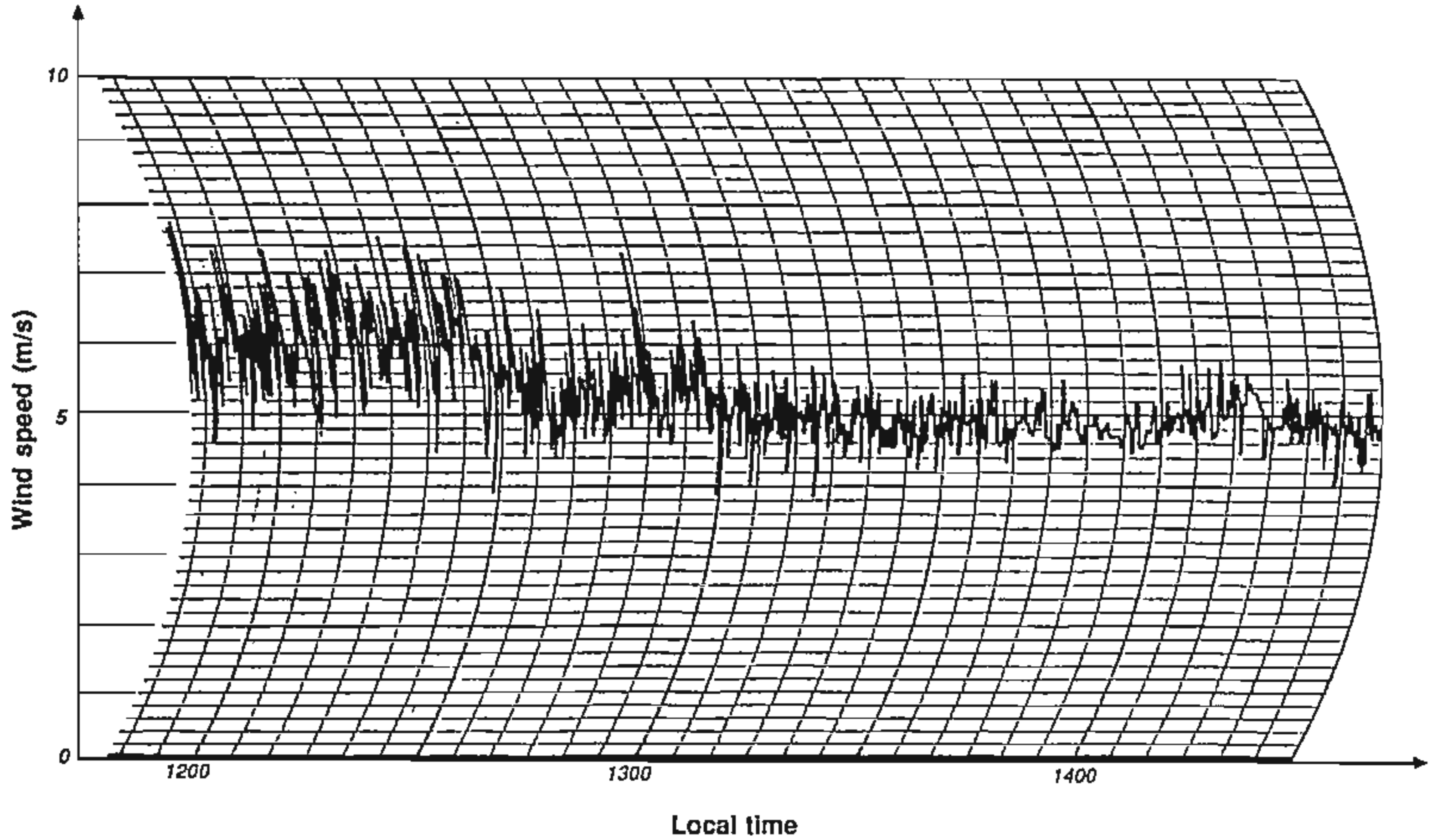

Fig. 2.1 Trace of wind speed obseved in early afternoon.

Instructions: go to menti.com enter code: 11564893

Then reply to the question: observe the signal, what can you say about it?

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Turbulence and its spectrum?

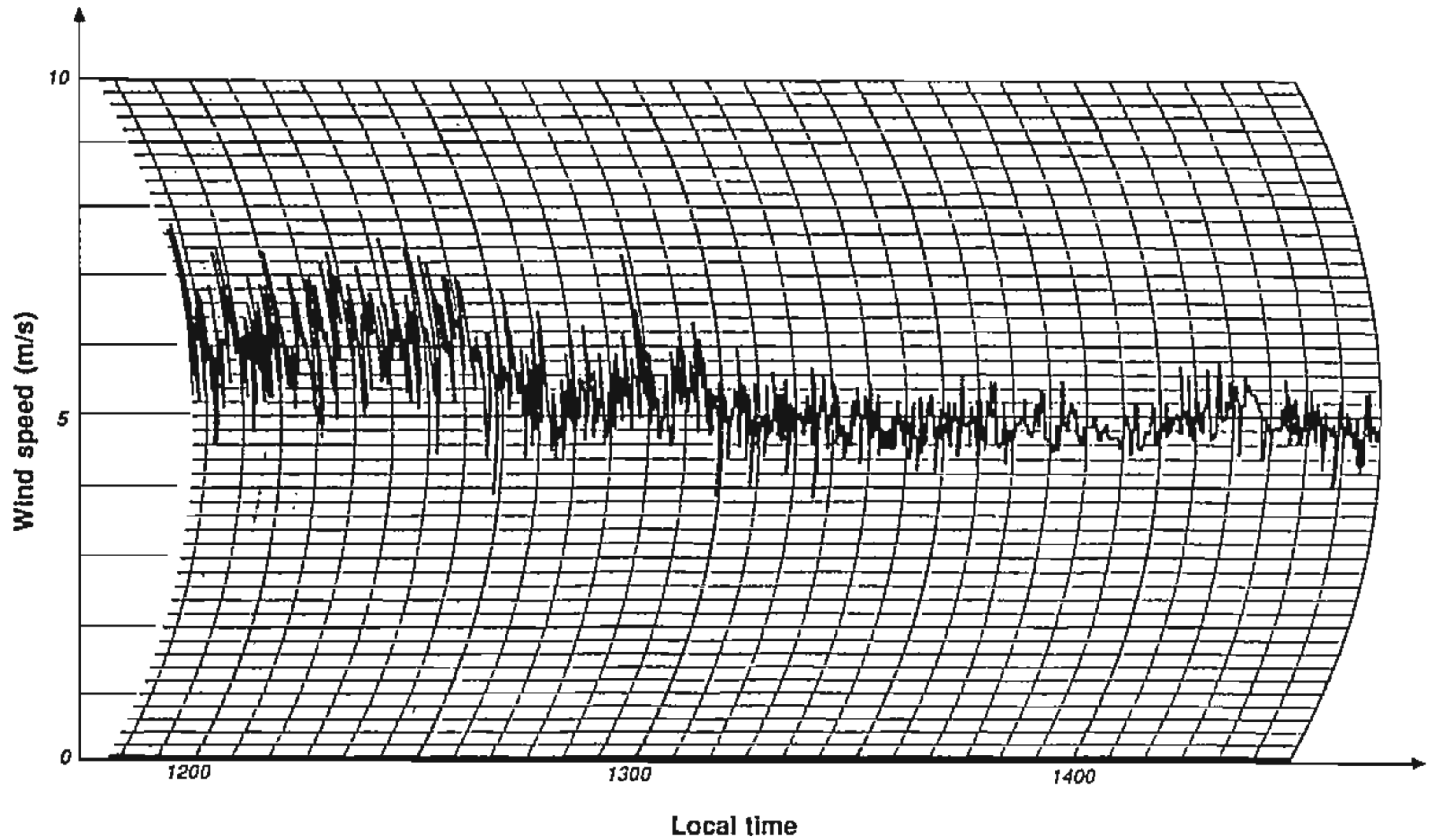

Fig. 2.1 Trace of wind speed obseved in early afternoon.

- Wind speed changes irregularly, quasi-randomly, which differentiates turbulence flow from waves
- There is limited ranges of speed, with a minimum and a maximum value. The amplitude of the signal is actually larger at the beginning than at the end of the signal, so it might be related to a more turbulent phenomena.
- From the signal, we can observe a mean value, or an average wind speed which is around 6m/s int he first part, and 5m/s in the last end of the signal

Questions so far?

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2. **Mean and turbulent parts**

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Mean and Turbulent Parts

We have a very easy way to isolate the large-scale variations from the turbulent ones from a wind speed signal:

- Over a finite time signal, we compute the mean value $$\overline{U} = \frac{1}{N}\sum_{t=0}^{N-1} U(t) \qquad N = samples\ in\ time$$
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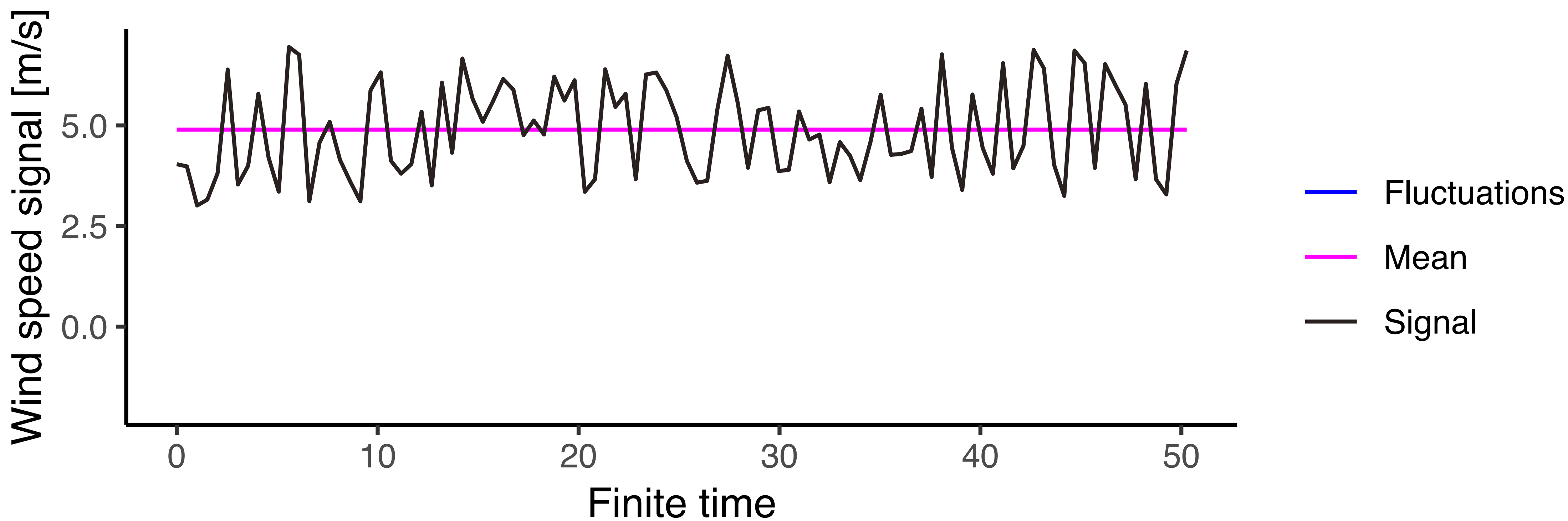

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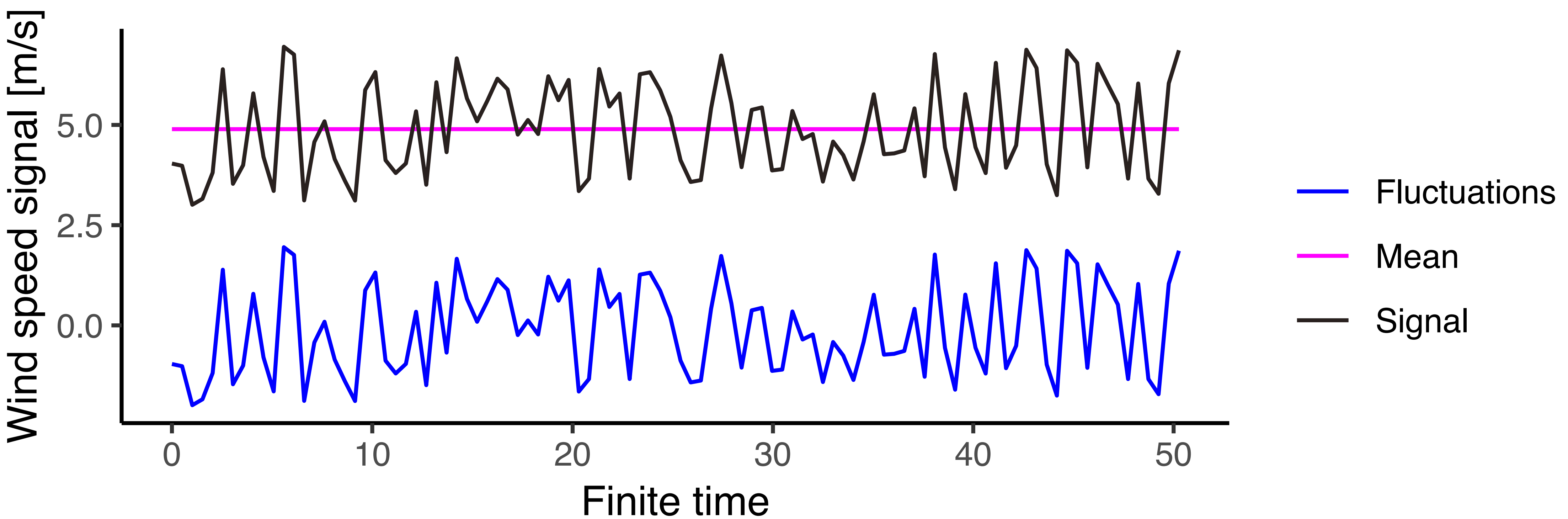

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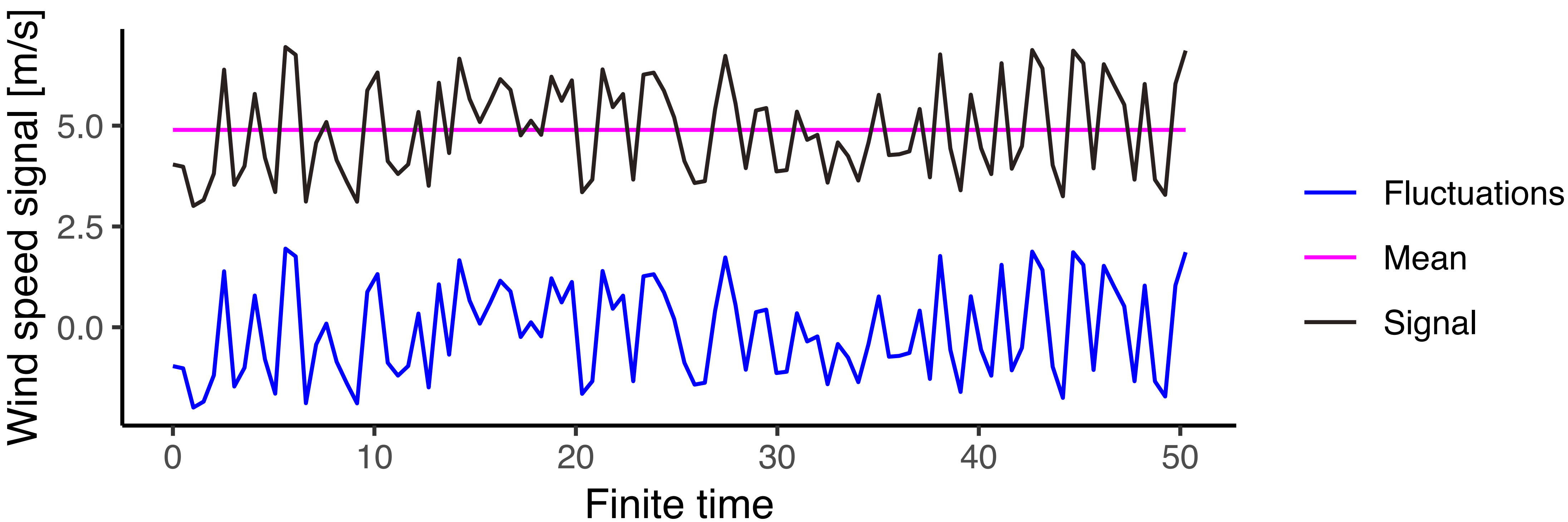

$$U = \overline{U} + u' \qquad V = \overline{V} + v' \qquad W = \overline{W} + w' \qquad \theta_v = \overline{\theta_v} + \theta_v' \qquad c = \overline{c} + c'$$

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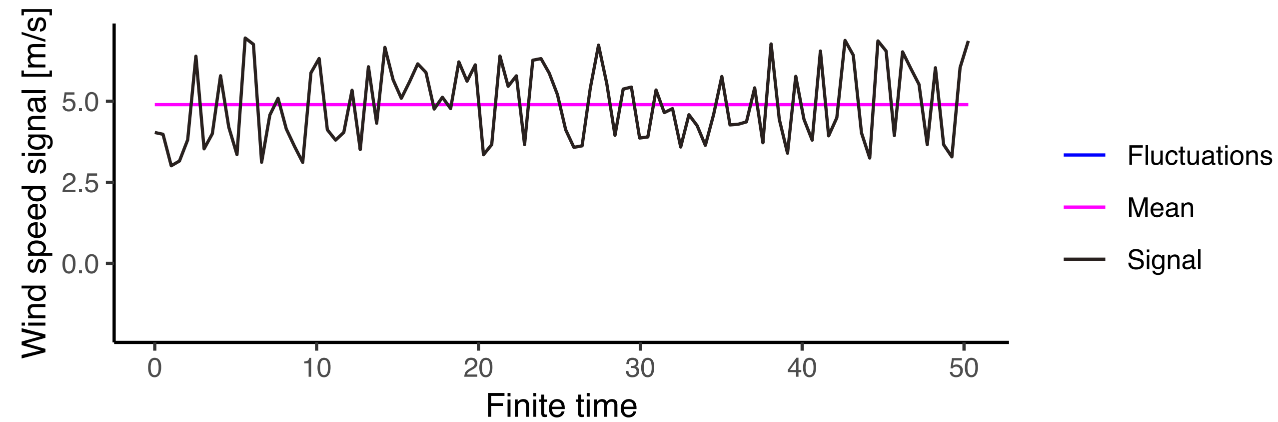

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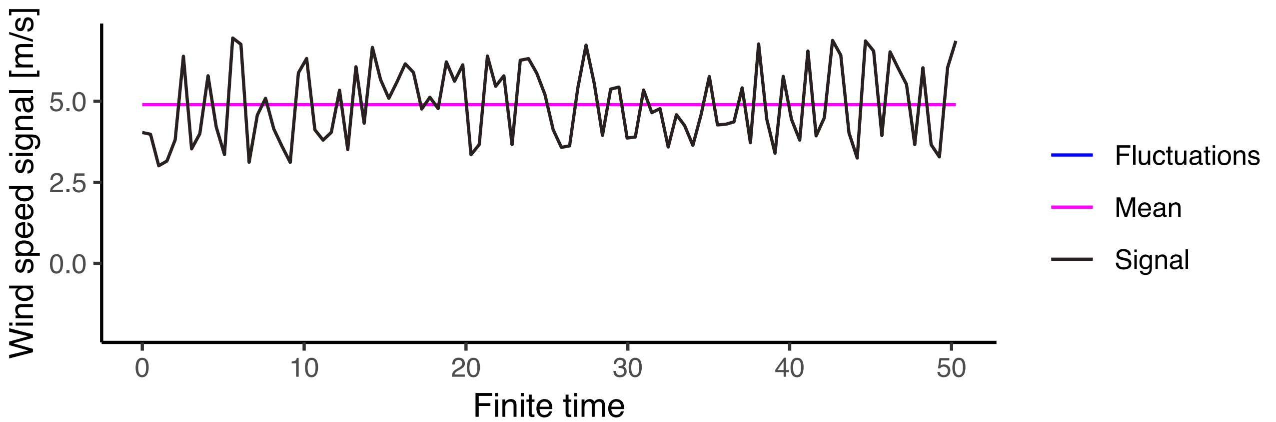

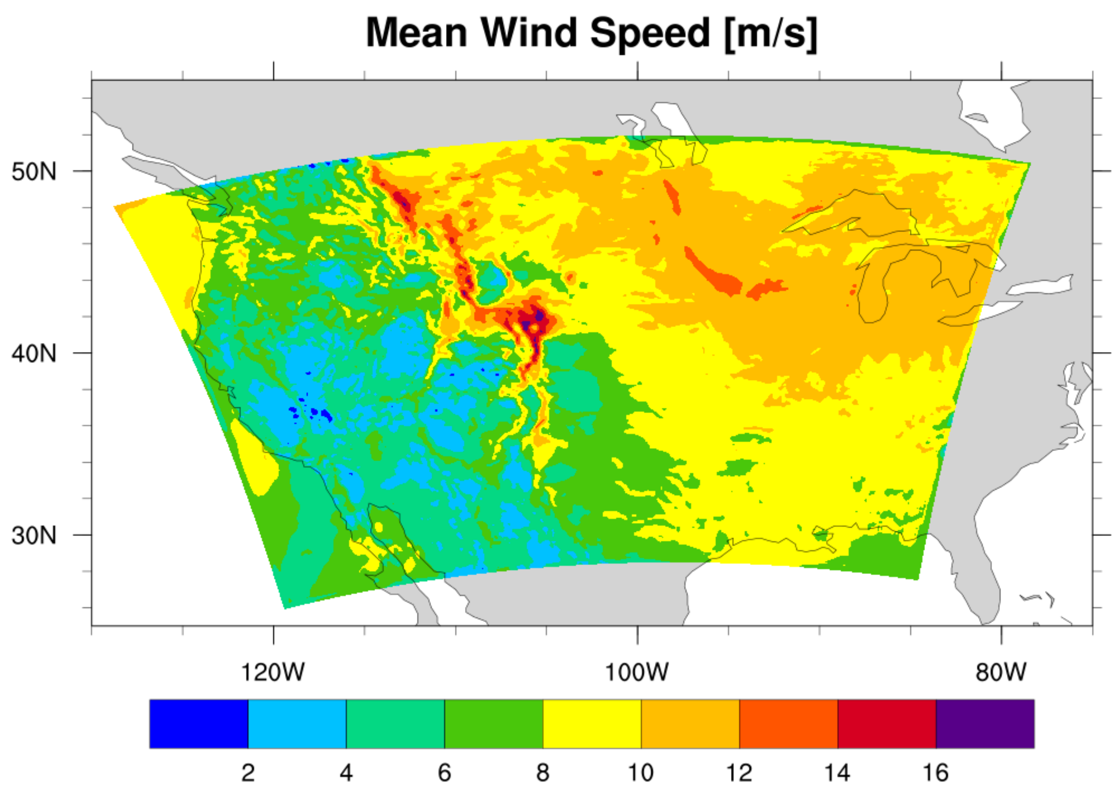

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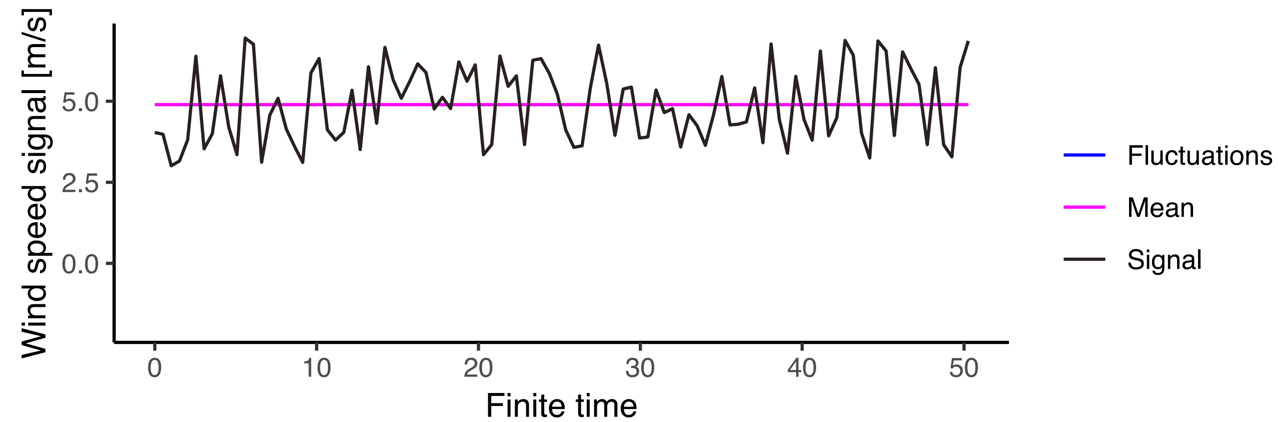

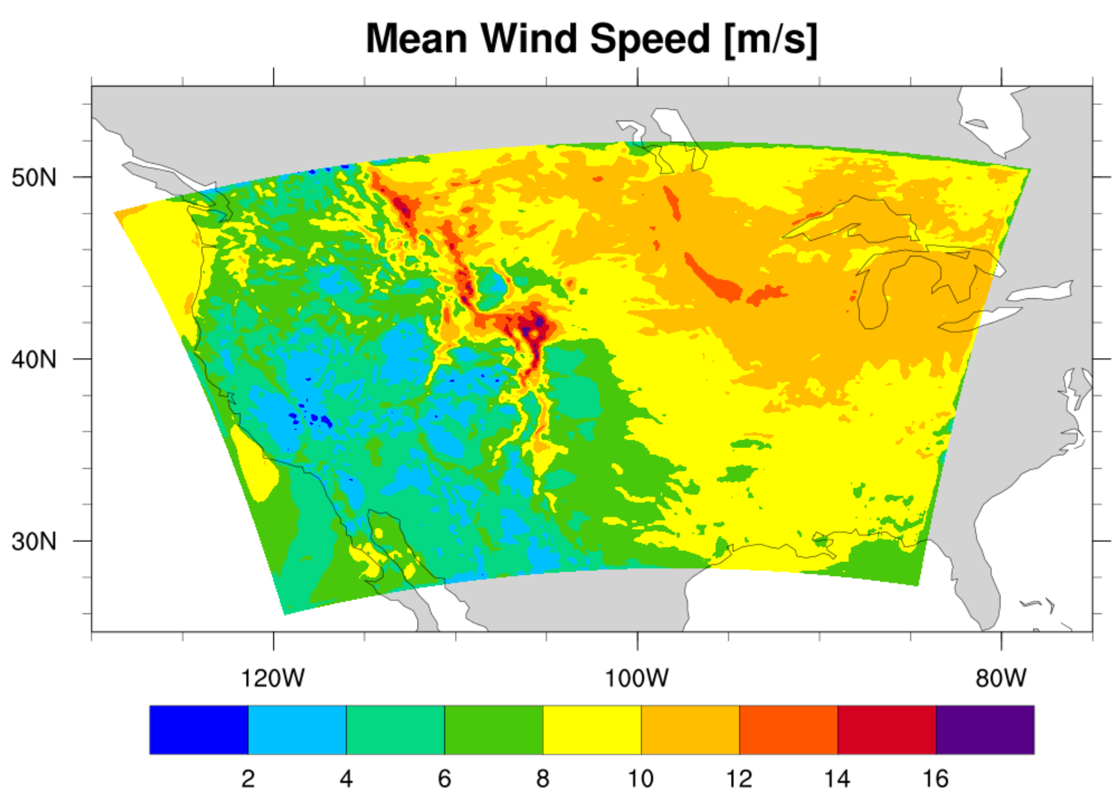

$$\overline{U} = \frac{1}{N_i} \frac{1}{N_j} \sum_{i=0}^{N_i-1} \sum_{j=0}^{N_j-1} U(i,j)$$

Some basic statistical methods

Rules of averaging

When we derive the mean flow equations in the following class, we use averaging rules.
A and B are two variables dependent of time (A(t), B(t)), and c is a constant.

[Fichtl, G., Boundary Layer Meteorology, 1971]

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Reynolds averaging

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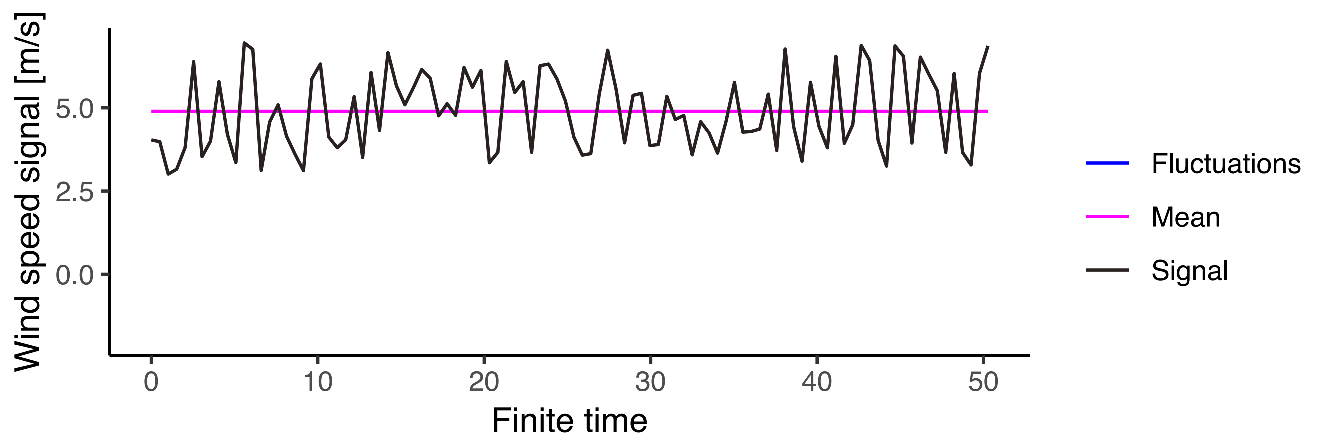

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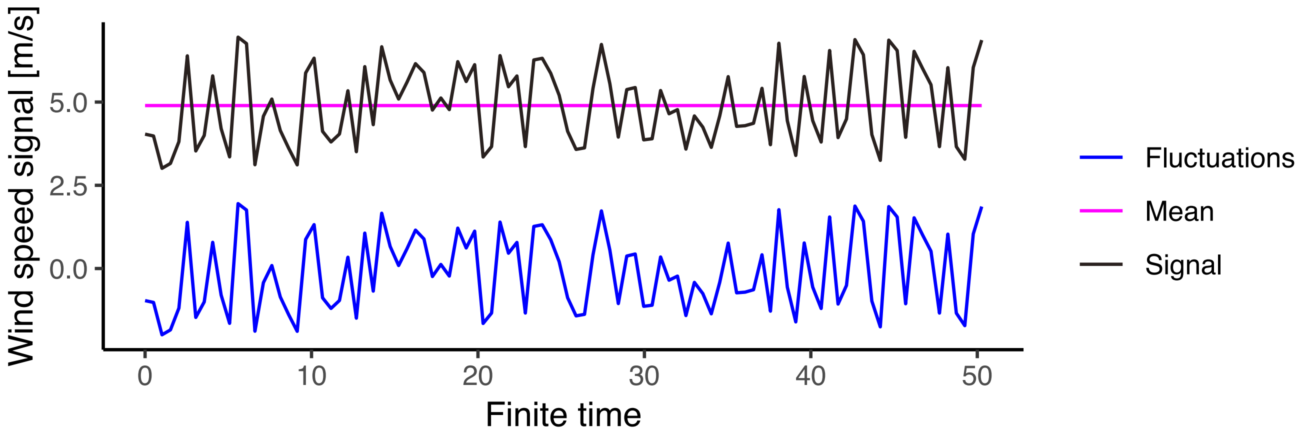

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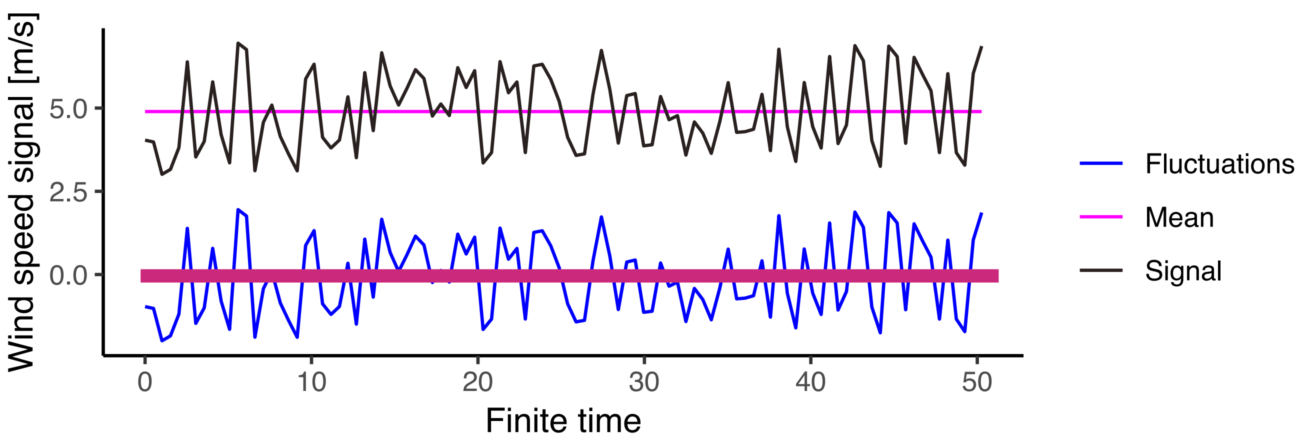

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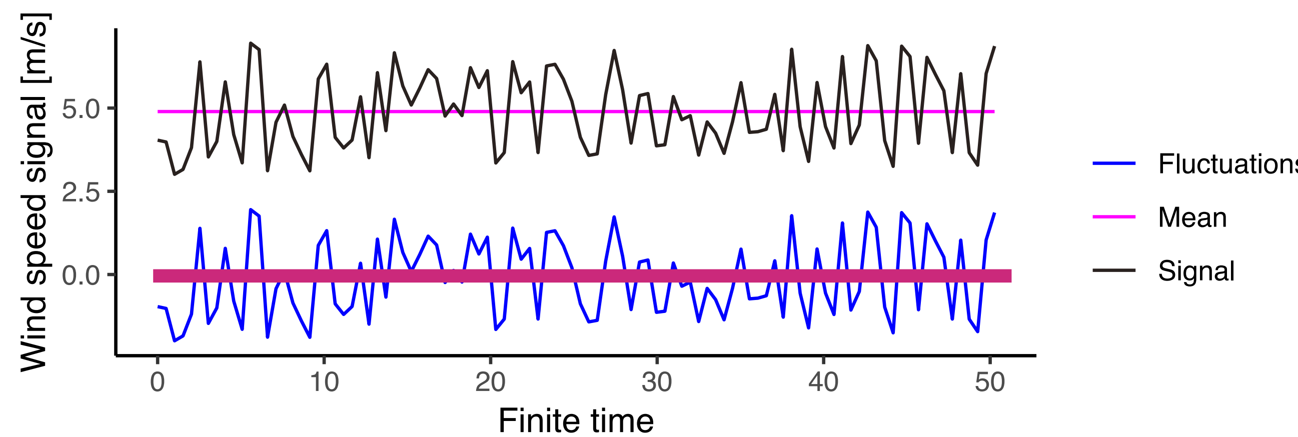

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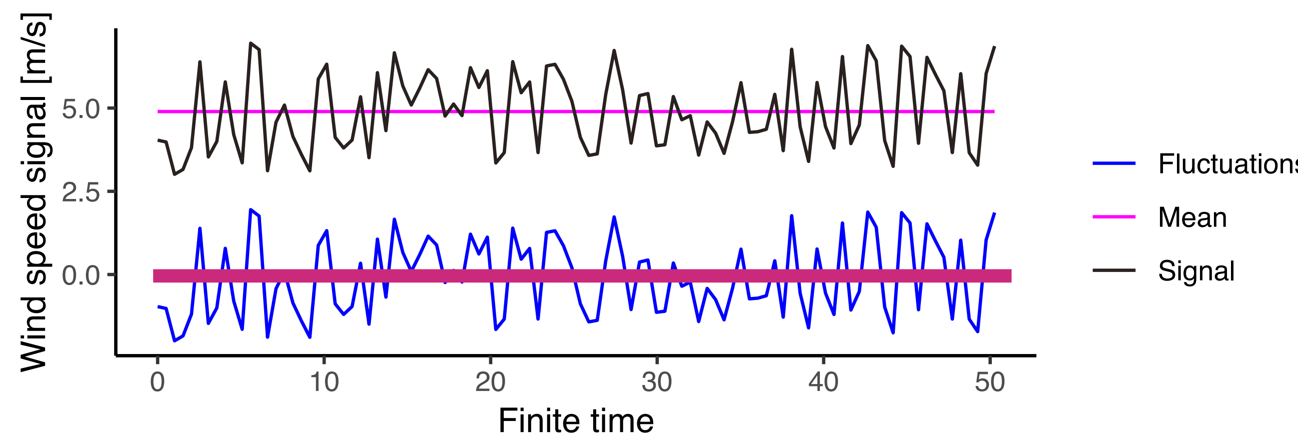

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Some basic statistical methods

Variance, standard deviation and turbulence intensity

$$\sigma_U^2 = \frac{1}{N}\sum_{t=0}^{N-1}\left(U(t) - \overline{U}\right)^2 \longrightarrow \sigma_U^2 = \frac{1}{N}\sum_{t=0}^{N-1} u'(t) = \overline{u'^2}$$

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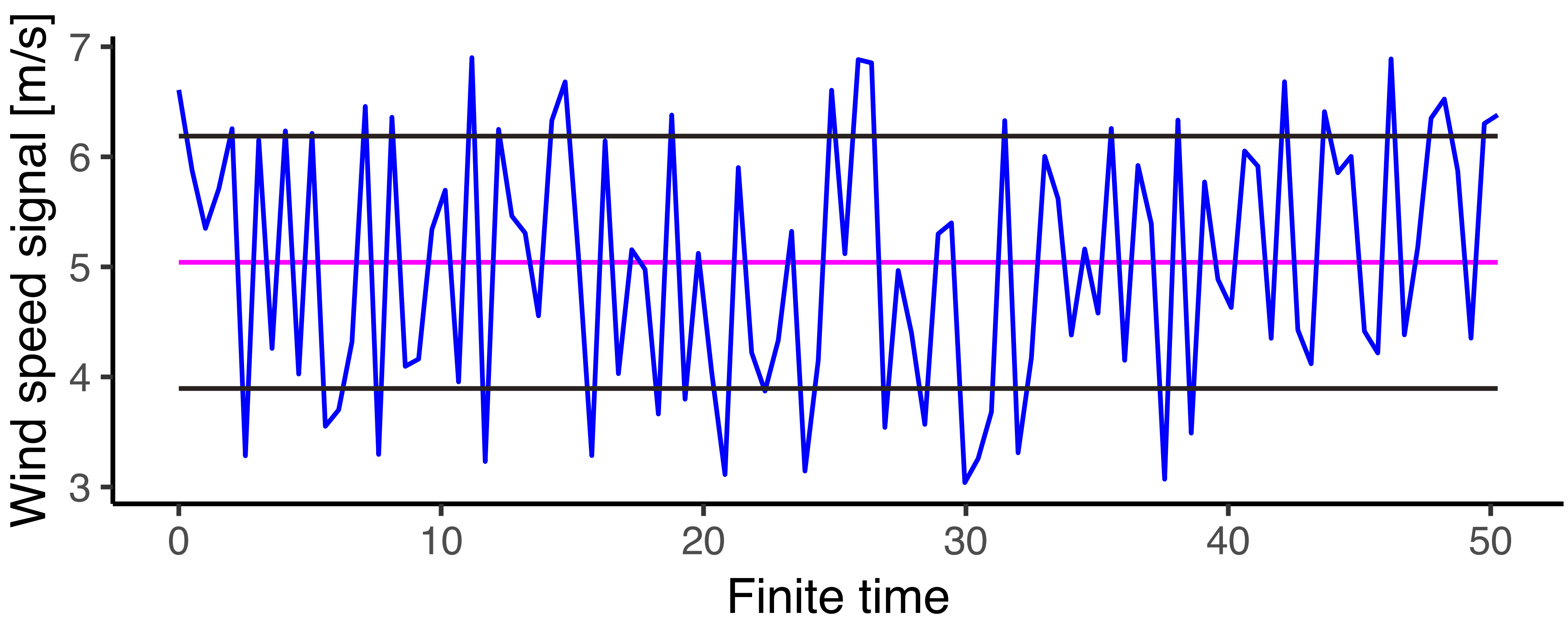

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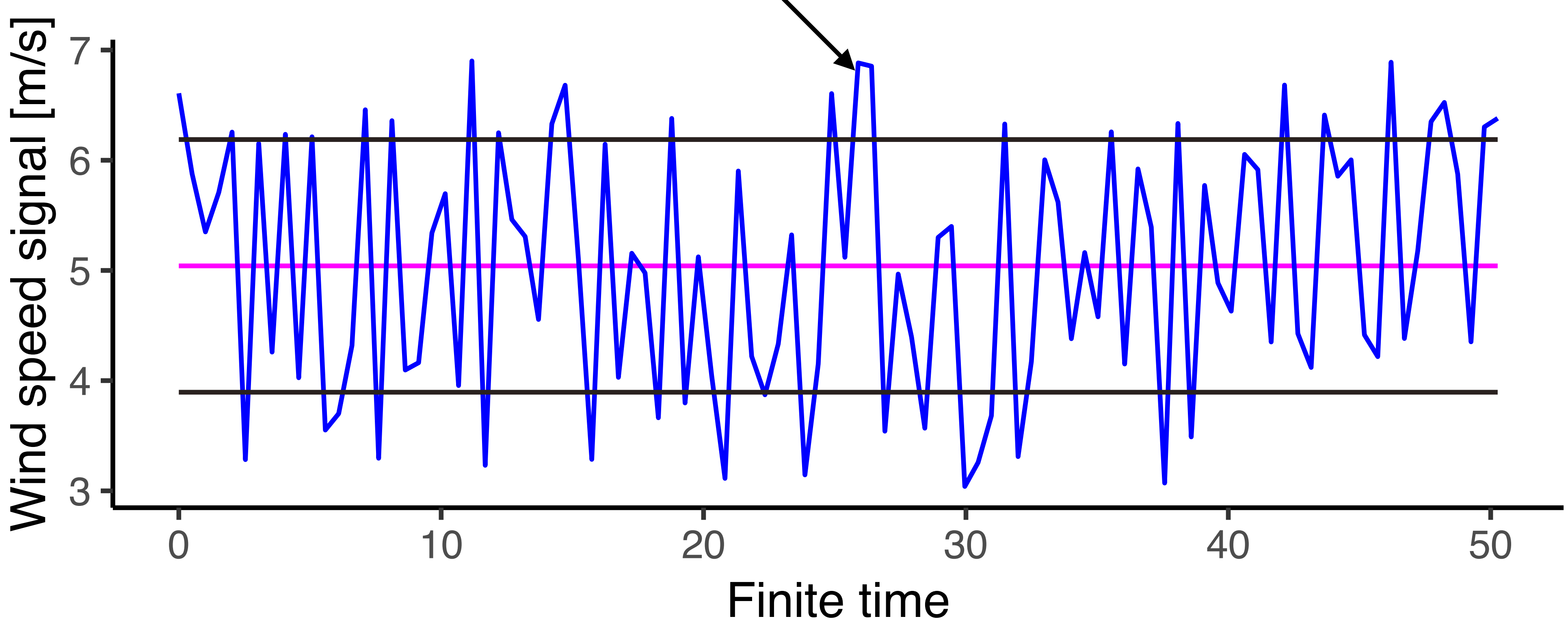

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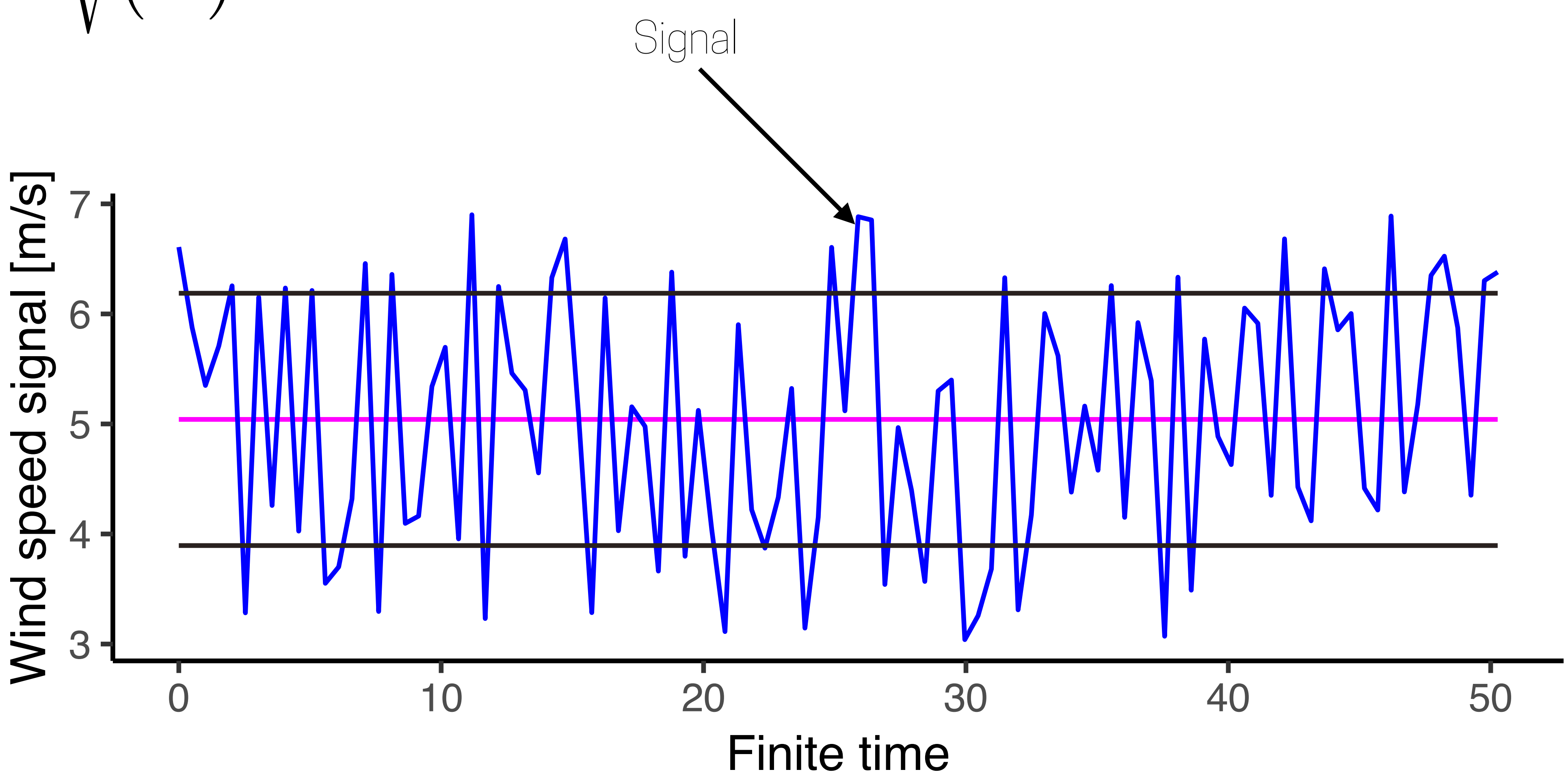

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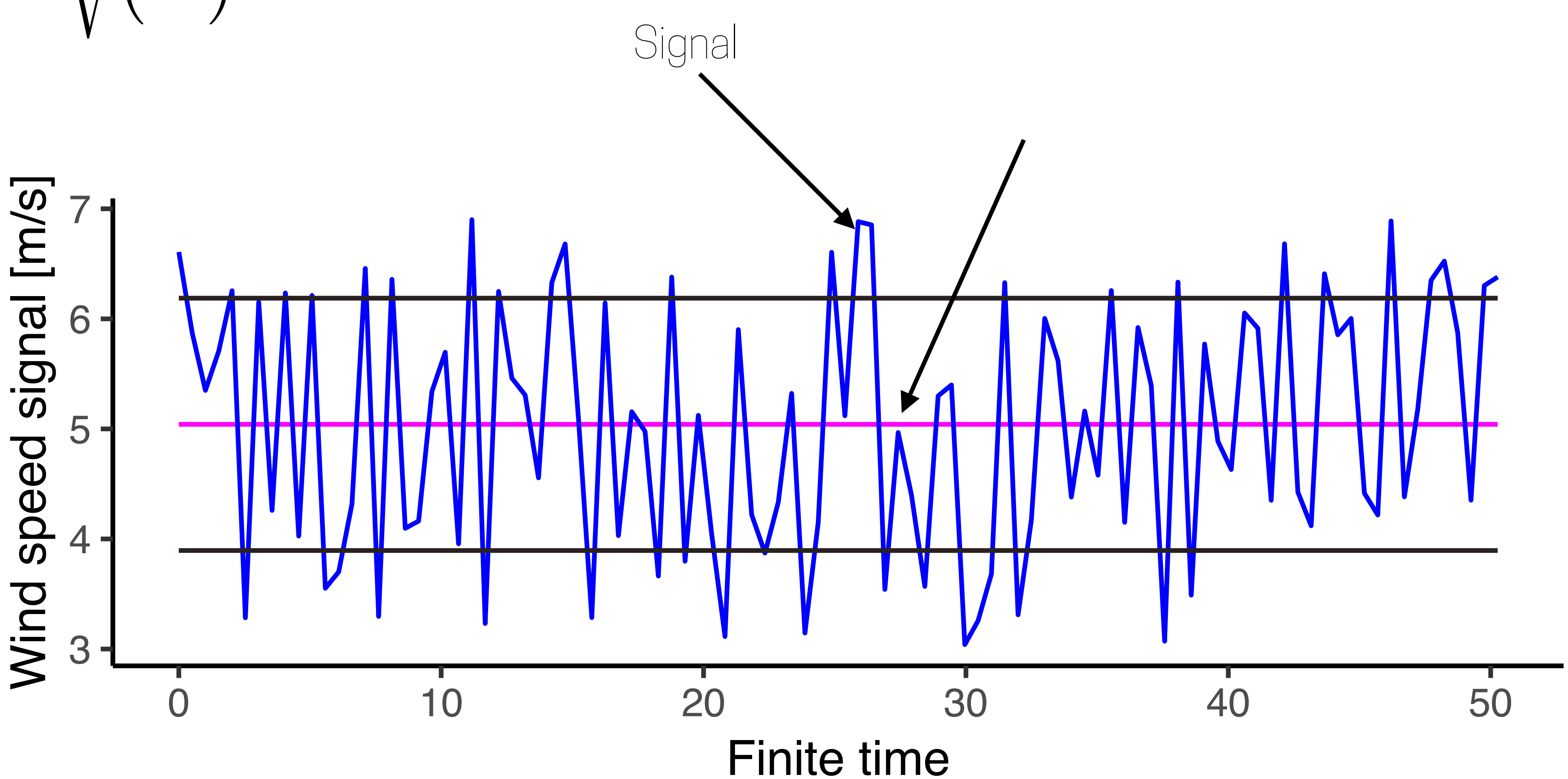

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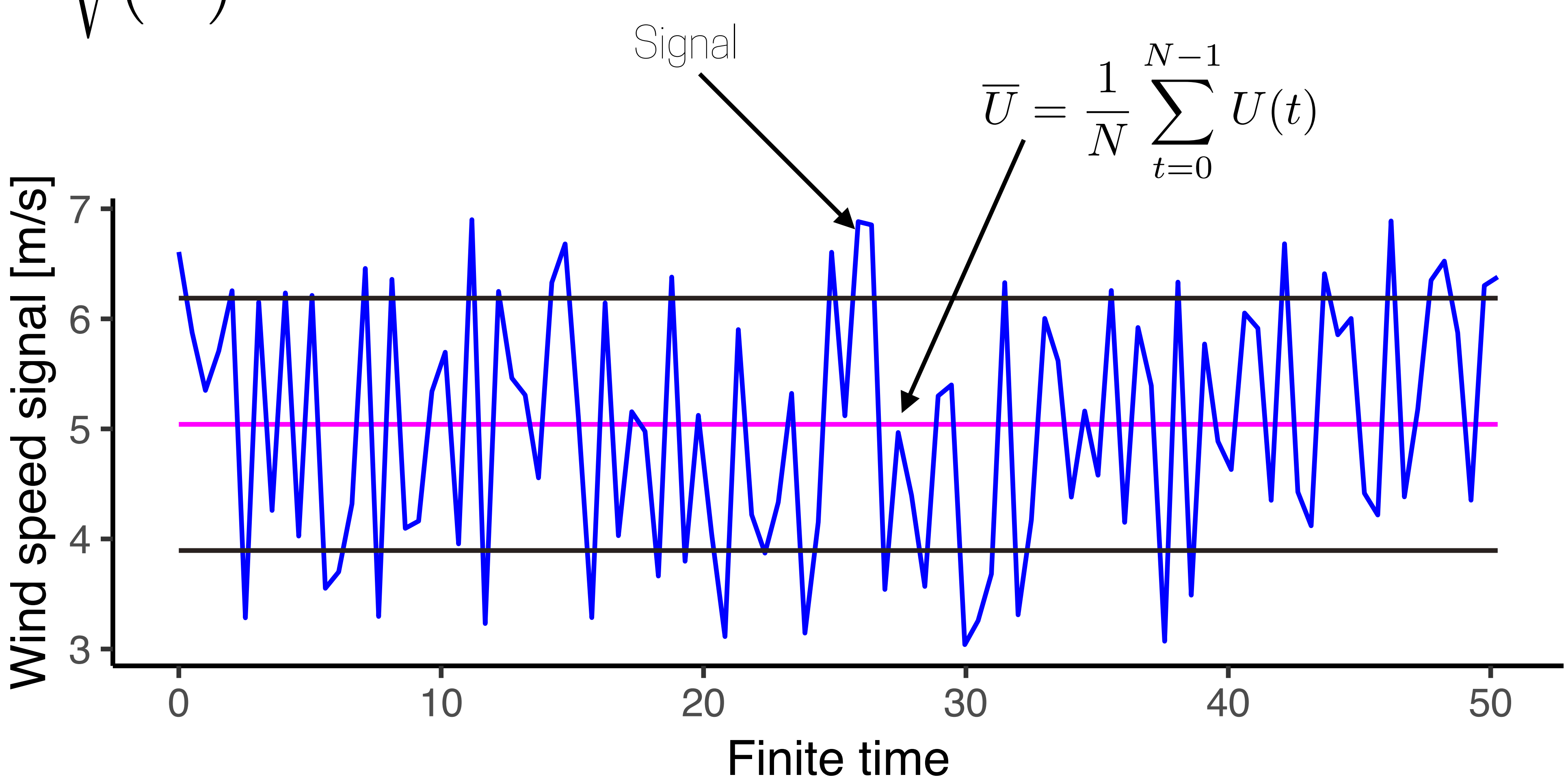

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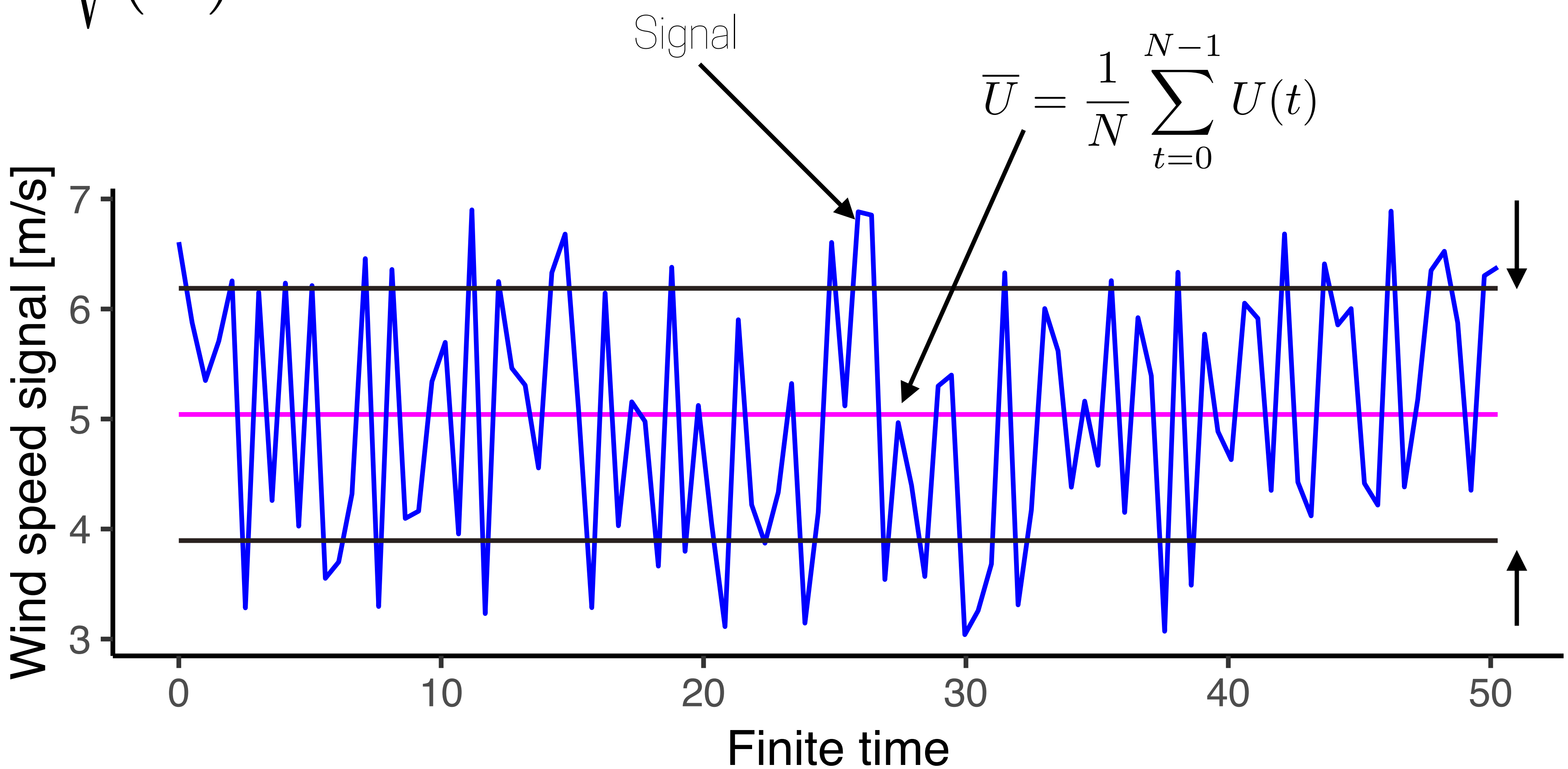

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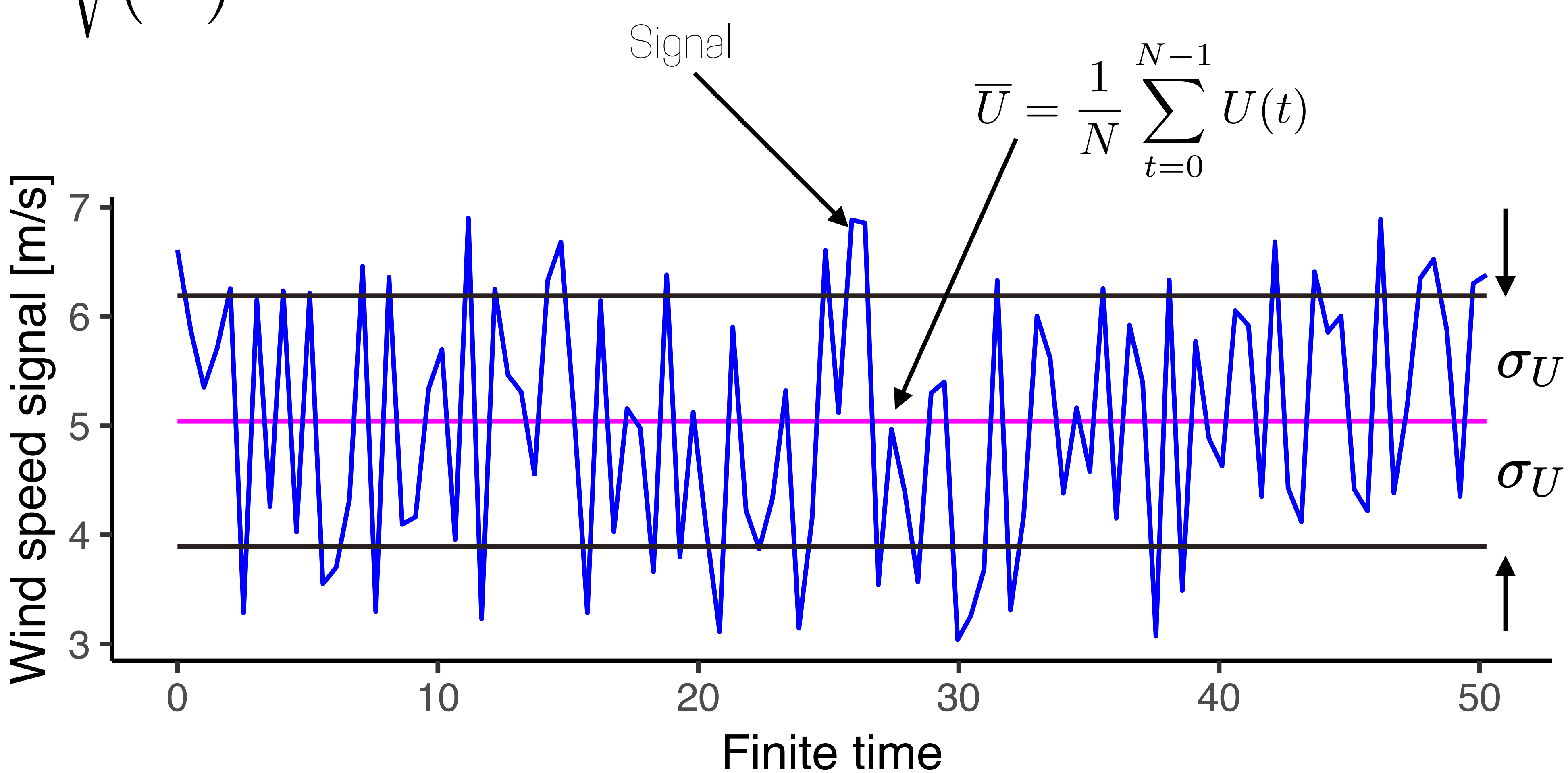

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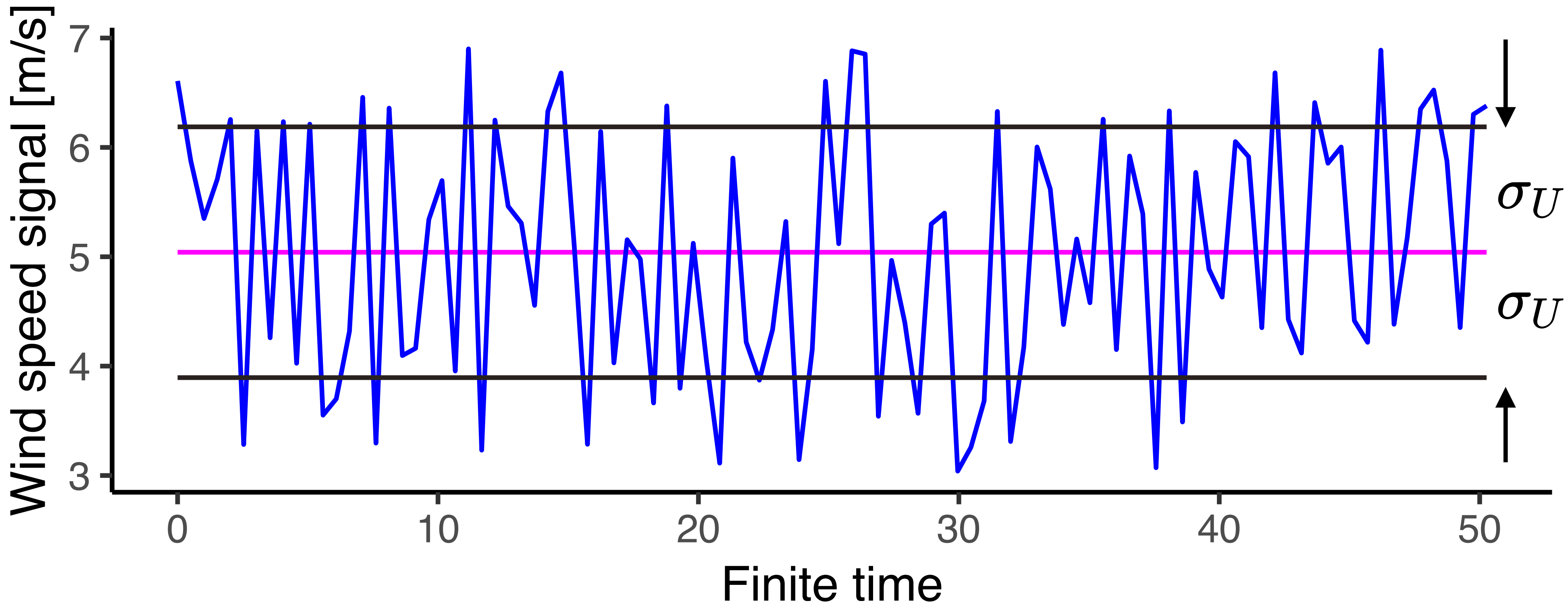

The standard deviation can be understood as a measure of the magnitude of the spread or dispersion of the original data from its mean; for this reason is used to measure turbulence intensity.

A dimensionless measure of **turbulence intensity**:

$$I = \frac{\sigma_U}{\overline{U}} = \frac{[m/s]}{[m/s]}$$

Some uses: Taylor hypothesis is only valid if I<0.5

Some basic statistical methods

Covariance and correlation

The **covariance** indicates the degree of common relationship between two variables, A and B. Its equation is the following:

$$
\begin{aligned}
covar(A,B) &= \frac{1}{N}\sum_{t=0}^{N-1}(A_t - \overline{A})(B_t - \overline{B}) \\
&= \frac{1}{N}\sum_{t=0}^{N-1}(a'_t)(b'_t) \\
&= \overline{a'b'}
\end{aligned}
$$

An example could be temperature, T, and vertical velocity, w. On a hot summer day over land, we might expect the warmer air to raise (T'>0, w'>0), and the cooler air to sink (T'<0,w'<0).

Thus the product overall would be:

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Thus the product overall would be: $\boldsymbol{T'w'} > \mathbf{0}$ $\longrightarrow$ This means that T and w vary together

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Some basic statistical methods

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Covariance and correlation

Correlation is a special kind of covariance, ranging between [-1,1]:

$$r_{AB} \equiv \frac{\overline{a'b'}}{\sigma_A \sigma_B}$$

Some basic statistical methods

Covariance and correlation

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When $r_{AB}=0$ → A,B have no relation

Questions so far?

Outline of the class:

1. Turbulence and its spectrum
2. Mean and turbulent parts
3. Some basic statistical methods
4. **Turbulent kinetic energy**
5. Flux and stress
6. Wrap-up

Turbulence Kinetic Energy

Typically kinetic energy is commonly defined as: $$KE = \frac{1}{2}mU^2$$

where m is the mass and U the speed of the fluid. However, when dealing with air, it is more convenient to talk about kinetic energy per unit mass: $$KE = \frac{1}{2}U^2$$

Separating U into mean (<U>) and turbulent U (u'), we can retrieve wind kinetic energy (MKE) and wind turbulence kinetic energy (e or k) per unit mass:

$$MKE = \frac{1}{2}\left(\overline{U}^2 + \overline{V}^2 + \overline{W}^2\right) \qquad e = \frac{1}{2}\left(u'^2 + v'^2 + w'^2\right)$$

Since rapid variation in the value of e can happen in time, we can average these instantaneous values, and derive a mean turbulence kinetic energy (TKE), which is more representative of the overall flow turbulence:

$$TKE = \frac{1}{2}\left(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}\right) = \overline{e}$$

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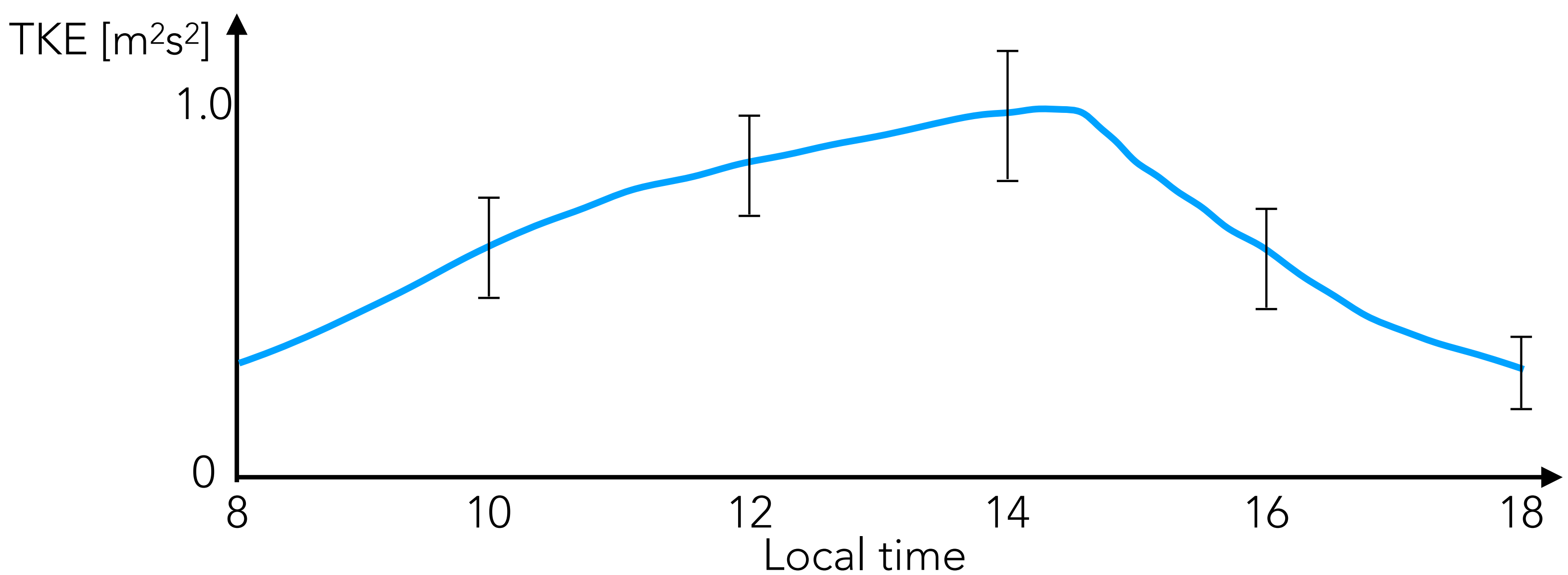
[Fichtl, G., Boundary Layer Meteorology, 1971]

Turbulence Kinetic Energy

- Turbulence kinetic energy is one of the most important quantities used to study the ABL
- By writing a budget equation for TKE, we can balance the production and loss terms, and determine whether the BL will become more or less turbulent



Typical day time variation of TKE, measured by aircraft below 300m in Tennessee, Bean and Repolf, 1983

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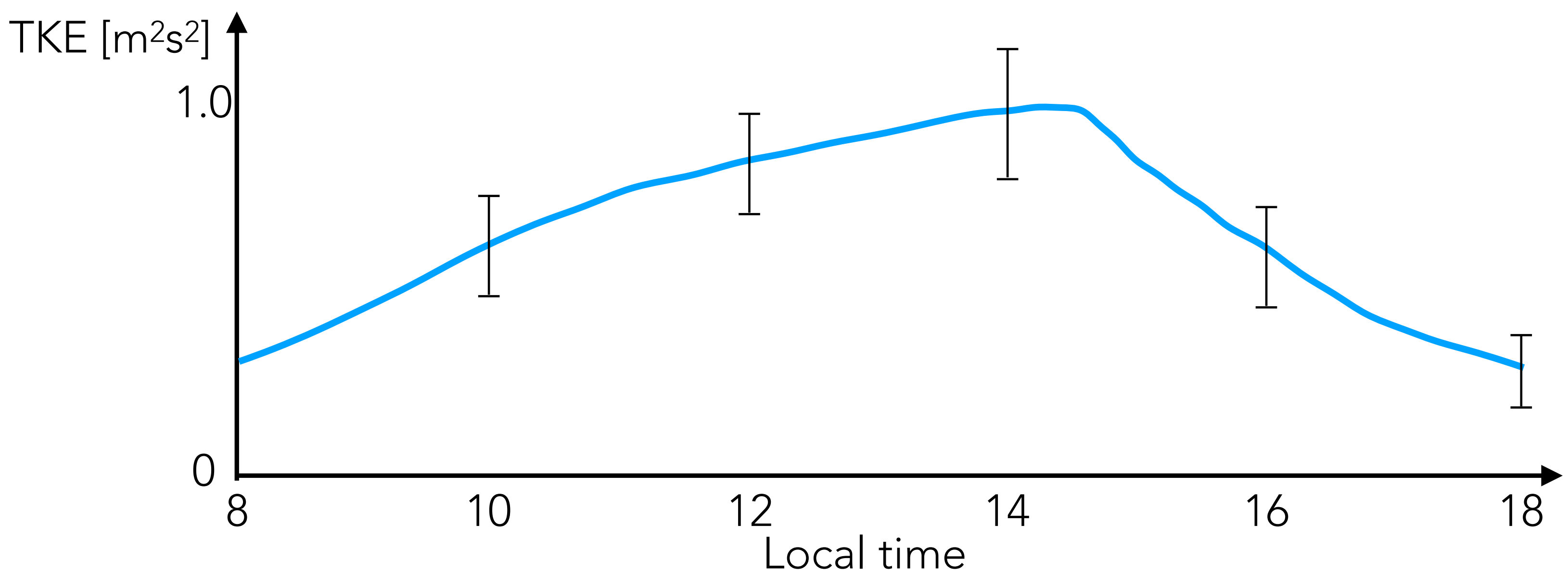

Typical day time variation of TKE, measured by aircraft below 300m in Tennessee, Bean and Repolf, 1983

- During sunny daytime, buoyancy allows air parcels to accelerate, allowing $\overline{w'^2}$ to be large and contribute to the TKE budget
- On overcast days, when there is little ground heating, wind shears and flow over obstacles create turbulence near the ground, that gradually decreases with height and it is primarily produced by and $\overline{u'^2}$ $\overline{v'^2}$
- At night TKE decays

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Flux and Stress

Flux: is the transfer of a quantity per unit area per unit time.

In the ABL, we normally talk about mass, heat, moisture, momentum and pollutant fluxes. The units is the variable units/m^2s: kg/m^2s (mass flux)

Stress: it is the force tending to produce deformation in a body, its units is force per unit area N/m^2, and in the atmosphere there are three types of stress: pressure, Reynolds stress and viscous stress

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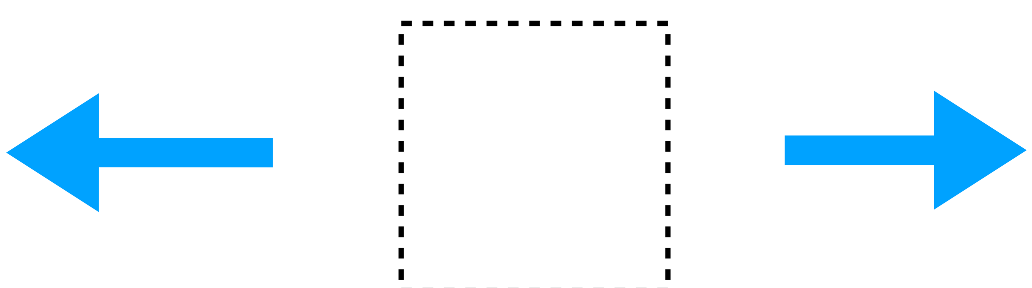

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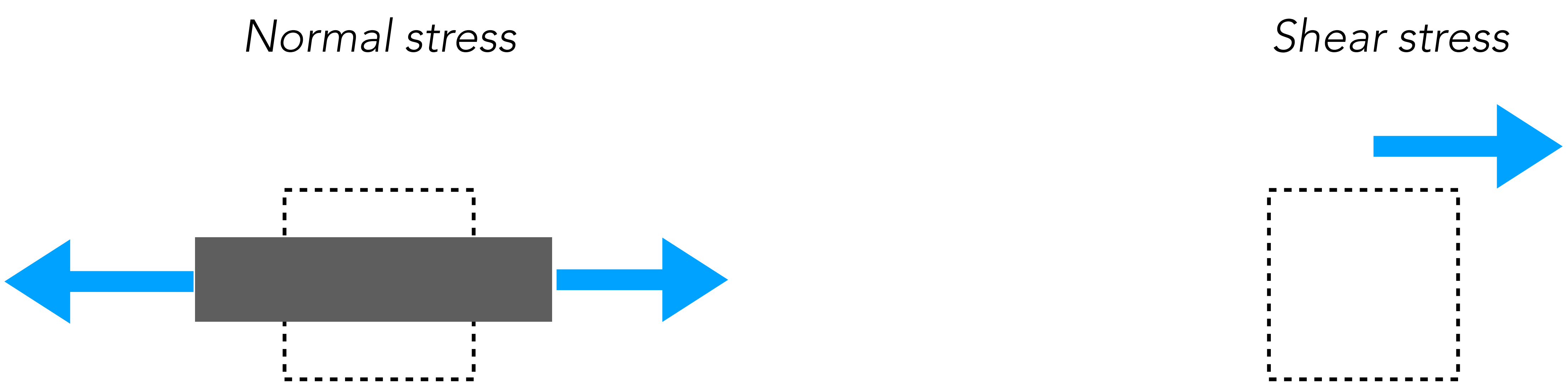

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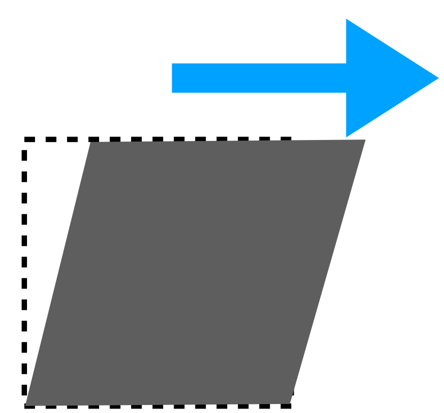

Flux and Stress

Fluid particles, you could think of small cubes, suffer similar stresses to solid objects

<u>Pressure:</u> is a type of stress that can act on a fluid at rest. Pressure acts equally in all directions, what is called isotropic. You can experience when you climb a mountain.

<u>Reynolds stress:</u>

Exists only in when the fluid is in turbulent motion

A turbulent eddy can mix air of different wind speeds into the cube of interest, and when different-speed air is added into one face of the cube, and not the opposite, the cube deforms between those two faces

Turbulent momentum flux acts like a stress, called Reynolds stress
Considering the 3D velocity components, its form is a symmetric tensor (6 components)

$$\begin{pmatrix} \overline{u'u'} & \overline{u'v'} & \overline{u'w'} \\ \overline{v'u'} & \overline{v'v'} & \overline{v'w'} \\ \overline{w'u'} & \overline{w'v'} & \overline{w'w'} \end{pmatrix}$$

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Flux and Stress

Fluid particles, you could think of small cubes, suffer similar stresses to solid objects

<u>Pressure:</u> is a type of stress that can act on a fluid at rest. Pressure acts equally in all directions, what is called isotropic. You can experience when you climb a mountain.

<u>Viscous stress:</u>

Exist when there is shearing motions in the fluid, no matter laminar or turbulent

As a portion of the fluid moves, the intermolecular forces tend to drag the adjacent fluid molecules in the same direction.

The components form also a 6 independent component tensor

A fluid for which the viscous stress is linearly dependent on the shear is said to be Newtonian

$$\tau_{ij} = \mu \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right) + \left(\mu_B - \frac{2}{3}\mu \right) \frac{\partial u_k}{\partial x_k} \delta ij$$

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Application real data

Application with real data

Given the following instantaneous measurements of potential temperature (**θ_v**) and vertical velocity (w) in this table, could you fill the remaining blanks?

Measurements			Calculations					
Index	w	θ_v	w'	θ'_v	$(w')^2$	$(\theta'_v)^2$	$w\theta_v$	$w'\theta'_v$
0	0.5	295						
1	-0.5	293						
2	1.0	295						
3	0.8	298						
4	0.9	292						
5	-0.2	294						
6	-0.5	292						
7	0.0	289						
8	-0.9	293						
9	-0.1	299						
Average								

Can you verify the following?

$$\overline{w\theta_v} = \overline{w}\,\overline{\theta_v} + \overline{w'\theta'_v}$$

Application with real data - Solution

Given the following instantaneous measurements of potential temperature (**θ_v**) and vertical velocity (w) in this table, could you fill the remaining blanks?

Measurements			Calculations					
Index	w	θ_v	w'	θ'_v	$(w')^2$	$(\theta'_v)^2$	$w\theta_v$	$w'\theta'_v$
0	0.5	295	0.4	1	0.16	1	147.5	0.4
1	-0.5	293	-0.6	-1	0.36	1	-146.5	0.6
2	1.0	295	0.9	1	0.81	1	295	0.9
3	0.8	298	0.7	4	0.49	16	238.4	2.8
4	0.9	292	0.8	-2	0.64	4	262.8	-1.6
5	-0.2	294	-0.3	0	0.09	0	-58.8	0
6	-0.5	292	-0.6	-2	0.36	4	-146	1.2
7	0.0	289	-0.1	-5	0.01	25	0	0.5
8	-0.9	293	-1	-1	1	1	-263.7	1
9	-0.1	299	-0.2	5	0.04	25	-29.9	-1
Average	0.1	294	0	0	0,396	7.8	29.88	0.48

Can you verify the following?

$$\overline{w\theta_v} = \overline{w}\,\overline{\theta_v} + \overline{w'\theta'_v}$$

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Wrap up:

1. Apply mean averaging rules
2. Understand basic statistical mean, variance and correlation concepts

Next class:

Computational fluid dynamics applied to urban cases

Outline of the class:

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7. One-minute notes (the muddiest point)

The muddiest point

Instructions: go to menti.com enter code: 11564893

Then reply to the question: what is more unclear for you up to now?

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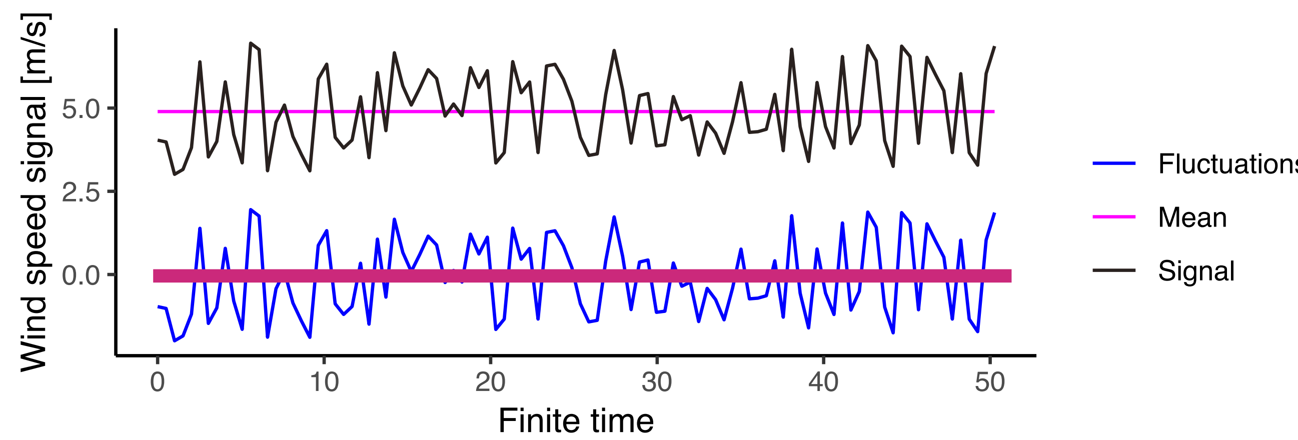
Some basic statistical methods

Reynolds averaging

The previous rules can now be applied to variables that are separated into mean and turbulent parts:

$$A = \overline{A} + a' \qquad B = \overline{B} + b'$$



$$\overline{(A)} = \overline{(\overline{A} + a')} = \overline{A} + \overline{a'} \rightarrow \overline{a'} = 0$$

$$\overline{(\overline{B}a')} = \overline{B}\,\overline{a'} = \overline{B}0 = 0$$

What about $\overline{(AB)}$?

Some basic statistical methods

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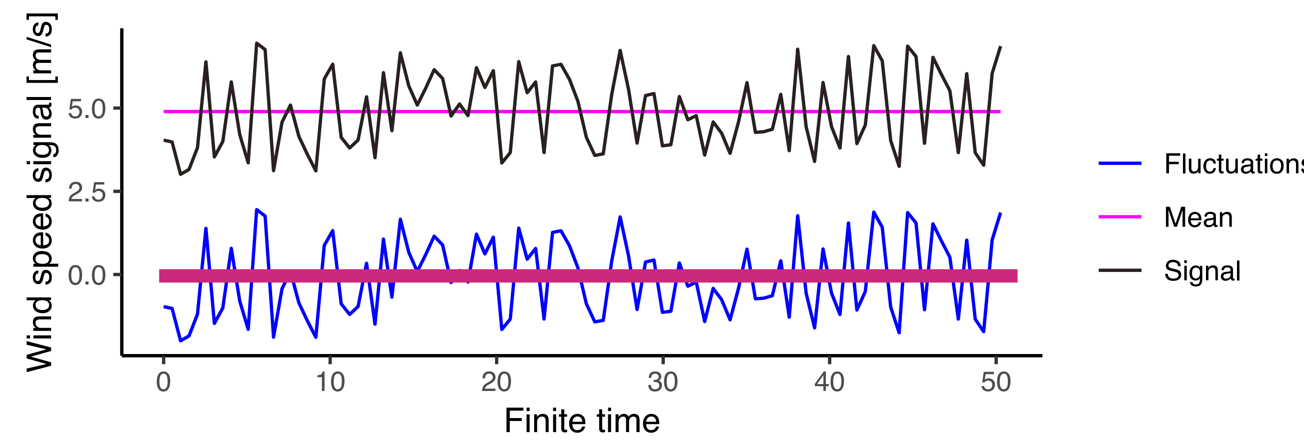

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$$\begin{aligned}\overline{(AB)} &= \overline{(\overline{A} + a')(\overline{B} + b')} \\ &= \overline{\overline{A}\overline{B} + a'\overline{B} + b'\overline{A} + a'b'} \\ &= \overline{(\overline{A}\overline{B})} + \overline{a'\overline{B}} + \overline{b'\overline{A}} + \overline{a'b'} \\ &= \overline{A}\overline{B} + \overline{a'b'}\end{aligned}$$